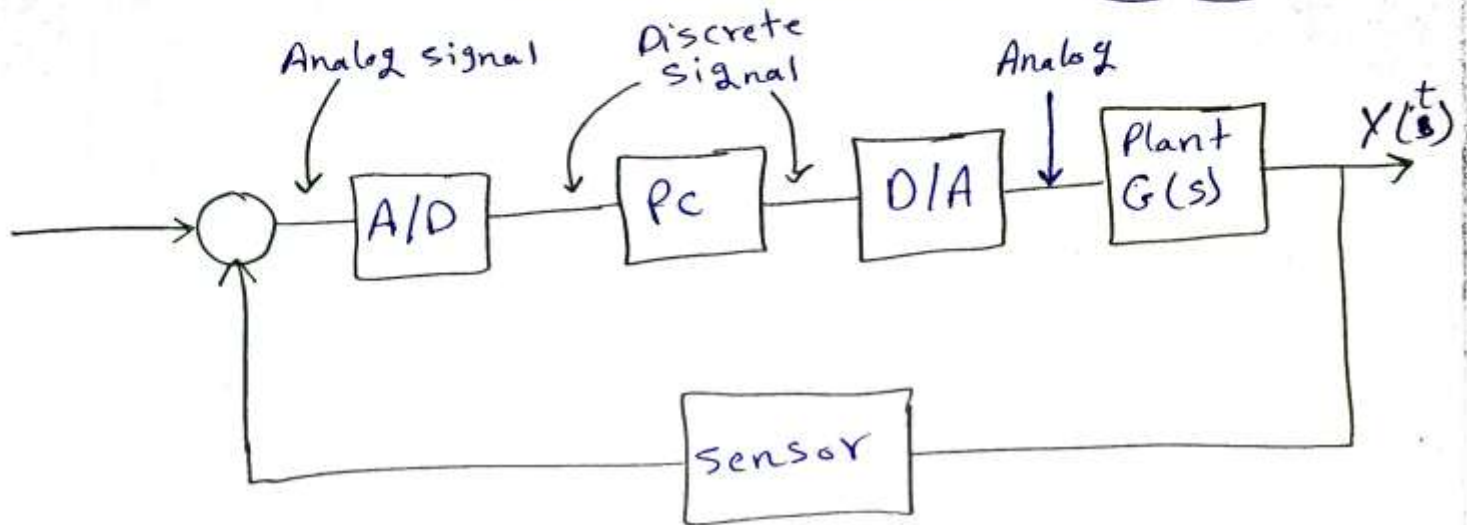


Lec 1

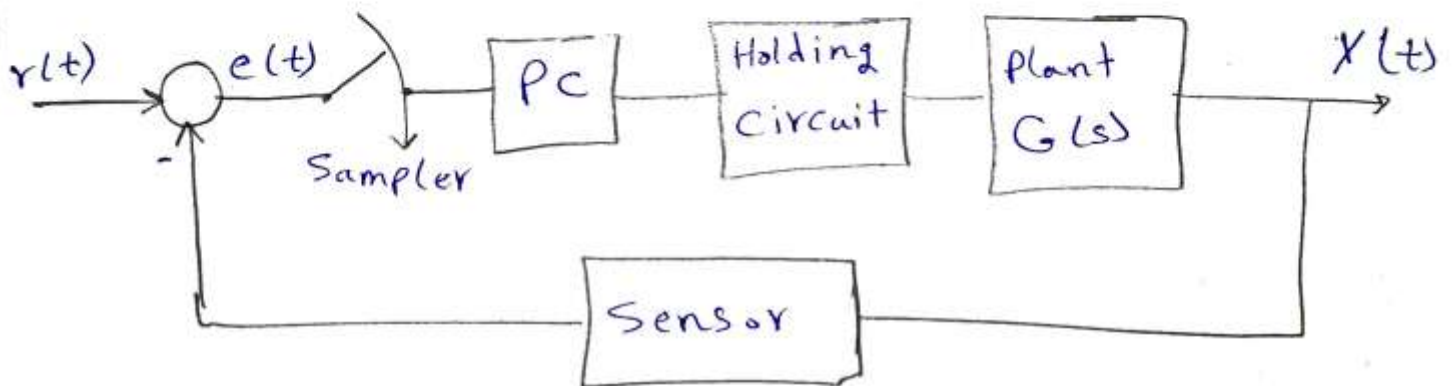
Revision on Z.T

Digital Control



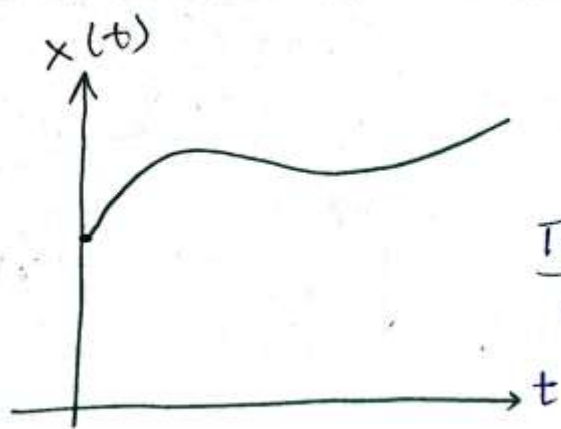
← معتمدين بالمشكل التالي

microControllers contains built in A/D Converter

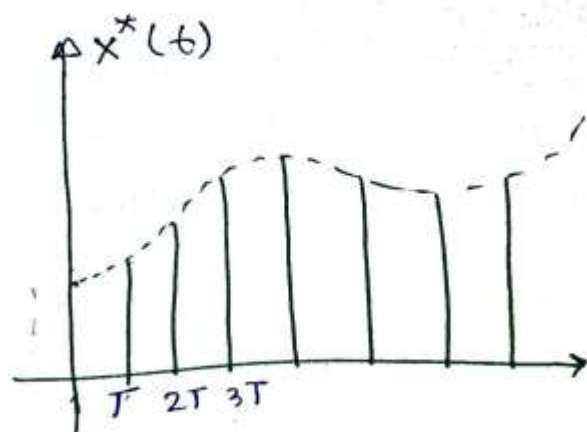


Digital Systems {
 1) A/D
 2) D/A Converter = holding circuit

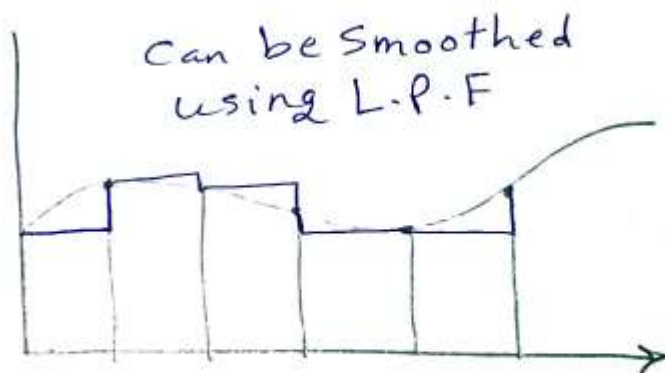
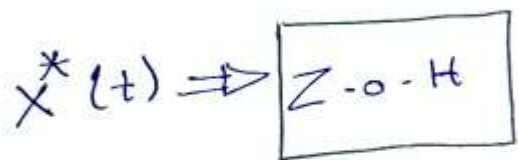
Holding ~~systems~~ circuit {
 Z-o-H (Zero order hold)
 F-o-H (First order hold)
 S-o-H (Second order hold)



$T = \text{Sampling Period}$

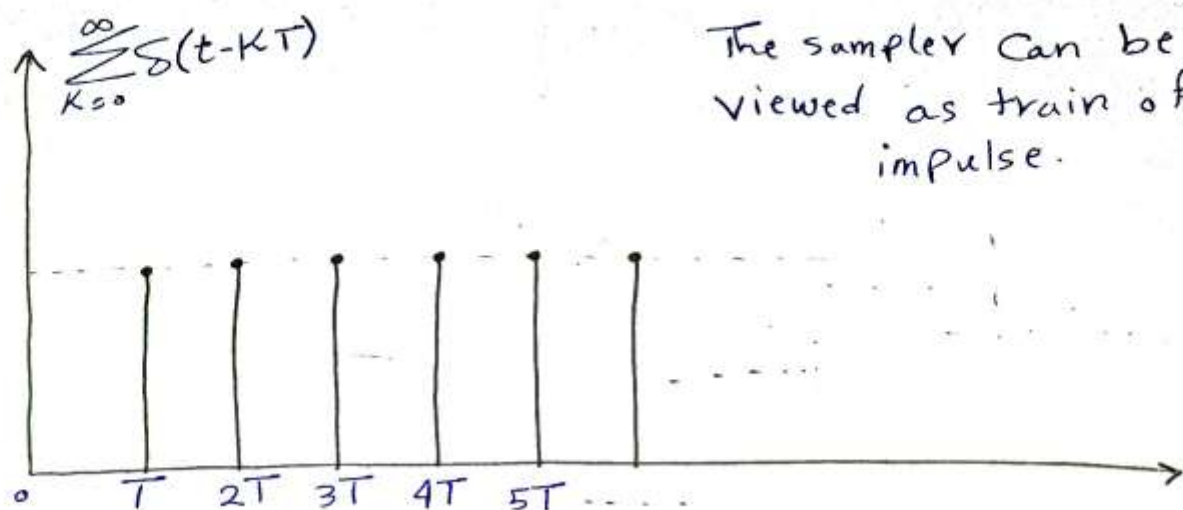


The output of the sampler is called star signal \equiv sampled signal



RC circuit that holds the signal value till the next value arrival

2



The sampler can be viewed as train of impulse.

output of sampler $x^*(t)$

$$x^*(t) = \sum_{k=0}^{\infty} x(t) \cdot \delta(t - kT) = x(0) + x(T) + x(2T) + \dots$$

where: $k \rightarrow$ sampling no.

$T \rightarrow$ sampling period.

L.T for $x^*(t) \Rightarrow x^*(t) = \sum_{k=0}^{\infty} x(kT) \delta(t - kT)$

$$X^*(s) = \sum_{k=0}^{\infty} x(kT) e^{-kTs}$$

\downarrow Z.T

$$X(z) = X^*(s) \Big|_{\substack{Ts \\ e^s = z}} = \sum_{k=0}^{\infty} x(kT) z^{-k}$$

$$x(t) \xrightarrow{Z.T} X(z) = \sum_{K=0}^{\infty} x(t) \Big|_{t=KT} z^{-K}$$

$$= \sum_{K=0}^{\infty} x(KT) z^{-K}$$

Ex:1 $x(t) = u(t) = 1$

$$X(z) = \sum_{K=0}^{\infty} 1 \cdot z^{-K} = \sum_{K=0}^{\infty} z^{-K}$$

$$= 1 + z^{-1} + z^{-2} + \dots = \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$$

Ex:2 $x(t) = e^{at}$

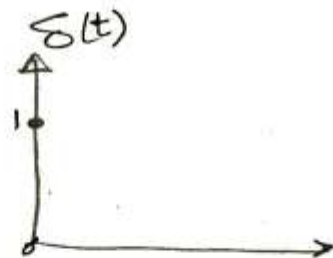
$$X(z) = \sum_{K=0}^{\infty} e^{aKT} z^{-K} = 1 + e^{aT} z^{-1} + e^{2aT} z^{-2} + \dots$$

$$= \frac{1}{1 - e^{aT} z^{-1}} = \frac{z}{z - e^{aT}}$$

Ex:3 $x(t) = \delta(t)$

$$X(z) = \sum_{K=0}^{\infty} \delta(KT) z^{-K}$$

$$= \delta(0) + \underbrace{\delta(T) z^{-1} + \dots}_{\rightarrow 0} = \delta(0) = 1$$



Ex: 4

$$x(t) = a^t$$

$$X(z) = \sum_{K=0}^{\infty} a^K T z^{-K} = 1 + a^T z^{-1} + a^{2T} z^{-2} + \dots$$

$$= \frac{1}{1 - a^T z^{-1}} = \frac{z}{z - a^T}$$

Ex: 5 $x(t) = t$

$$X(z) = \sum_{K=0}^{\infty} K T z^{-K} = 0 + T z^{-1} + 2T z^{-2} + 3T z^{-3} + \dots$$

بالضرب في z

$$z X(z) = T + 2T z^{-1} + 3T z^{-2} + \dots$$

$$z X(z) - X(z) = T + T z^{-1} + T z^{-2} + T z^{-3} + \dots$$

$$(z-1) X(z) = \frac{T}{1 - z^{-1}} = \frac{Tz}{z-1}$$

$$X(z) = \frac{Tz}{(z-1)^2}$$

Ex: 6 $x(t) = \sin(\omega t)$

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$X(z) = \sum_{k=0}^{\infty} \sin(\omega k T) z^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{e^{j\omega k T} - e^{-j\omega k T}}{2j} \right) z^{-k}$$

$$= \frac{1}{2j} \left[\sum_{k=0}^{\infty} e^{j\omega k T} \cdot z^{-k} - \sum_{k=0}^{\infty} e^{-j\omega k T} \cdot z^{-k} \right]$$

$$= \frac{1}{2j} \left[\left(1 + e^{j\omega T} z^{-1} + e^{2j\omega T} z^{-2} + \dots \right) \right.$$

$$\left. - \left(1 + e^{-j\omega T} z^{-1} + e^{-2j\omega T} z^{-2} + \dots \right) \right]$$



$$= \frac{1}{2j} \left[\frac{1}{1 - e^{j\omega T} z^{-1}} - \frac{1}{1 - e^{-j\omega T} z^{-1}} \right]$$

$$= \frac{1}{2j} \left[\frac{z}{z - e^{j\omega T}} - \frac{z}{z - e^{-j\omega T}} \right]$$

$$= \frac{1}{2j} \left[\frac{z(z - e^{-j\omega T}) - z(z - e^{j\omega T})}{(z - e^{j\omega T})(z - e^{-j\omega T})} \right]$$

$$= \frac{z}{2j} \left[\frac{-e^{-j\omega T} + e^{j\omega T}}{z - z \left(\frac{e^{j\omega T} + e^{-j\omega T}}{2} \right) + 1} \right]$$

$$= \frac{z \sin(\omega T)}{z^2 - 2z \cos(\omega T) + 1}$$

Ex: 7) $x(t) = \cos(\omega t)$

$$X(z) = \frac{z(z - \cos(\omega T))}{z^2 - 2z \cos(\omega T) + 1}$$

$$\boxed{1} \quad Z \left[\left. f_1(t) \pm f_2(t) \right|_{t=KT} \right] = F_1(z) \pm F_2(z)$$

$$\boxed{2} \quad Z \left[\left. a f(t) \right|_{t=KT} \right] = a F(z)$$

$$\boxed{3} \quad Z \left[\left. e^{at} f(t) \right|_{t=KT} \right] = F(z) \Big|_{z \rightarrow z e^{aT}}$$

$$\boxed{4} \quad Z \left[\left. t f(t) \right|_{t=KT} \right] = F(z) \Big|_{z = \frac{z}{aT}}$$

$$\boxed{5} \quad Z \left[\left. t f(t) \right|_{t=KT} \right] = -TZ \frac{dF(z)}{dz}$$

6 initial value

$$f(0) = \lim_{t \rightarrow 0} f(t) = \lim_{z \rightarrow \infty} F(z)$$

7 Final value (value at $t = \infty$)

$$f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{z \rightarrow 1} (z-1) F(z)$$

$$F(\infty) = \lim_{z \rightarrow 1} (1 - z^{-1}) F(z)$$

7] shifting

1. Delay

$$Z[x(k-n)] = z^{-n} \underbrace{x(z)}_{\rightarrow Z[x(k)]}$$

2. Advance

$$Z[x(k+n)] = z^n x(z) - z^n x(0) - z^{n-1} x(1) \\ - \dots - z x(n-1)$$

For ex

$$Z[x(k+1)] = z x(z) - z x(0)$$

$$Z[x(k+2)] = z^2 x(z) - z^2 x(0) - z x(1)$$

$$Z[x(k+3)] = z^3 x(z) - z^3 x(0) - z^2 x(1) - z x(2)$$

$$Z[X(k+1)] = \sum_{k=0}^{\infty} X(k+1) Z^{-k}$$

$$= X(1) + X(2) Z^{-1} + X(3) Z^{-2} + \dots \rightarrow (1)$$

$$\therefore X(z) = \sum_{k=0}^{\infty} X(k) Z^{-k}$$

$$= X(0) + X(1) Z^{-1} + X(2) Z^{-2} + \dots$$

multiply (1) with $Z Z^{-1}$

$$Z[X(k+1)] = Z Z^{-1} [X(1) + X(2) Z^{-1} + X(3) Z^{-2} + \dots]$$

$$= Z [\cancel{Z^{-1} X(1)} + X(2) Z^{-2} + X(3) Z^{-3} + \dots]$$

$$= Z \left[\underbrace{X(0) + X(1) Z^{-1} + X(2) Z^{-2} + \dots}_{\rightarrow X(z)} - X(0) \right]$$

$$= Z X(z) - Z X(0) \quad \neq$$

10

$$\text{Ex: } y(k+2) + 3y(k+1) + 2y(k) = \delta(k)$$

if $y(0) = 0$, $y(1) = -1$, solve for $y(k)$

Z-T

$$z^2 Y(z) - z^2 y(0) - z y(1) + 3[z Y(z) - z y(0)]$$

$$+ 2 Y(z) = 1$$

$$z^2 Y(z) + z + 3z Y(z) + 2 Y(z) = 1$$

$$(z^2 + 3z + 2) Y(z) = 1 - z$$

$$Y(z) = \frac{1-z}{z^2 + 3z + 2} = \frac{1-z}{(z+1)(z+2)}$$

$$= \frac{A_1}{z+1} + \frac{A_2}{z+2} = \frac{2}{z+1} - \frac{3}{z+2}$$

$$= \frac{2z z^{-1}}{z+1} - 3 \frac{z z^{-1}}{z+2}$$

$\mathcal{I} \cdot \text{Z-T} \rightarrow (z^{-1} \cdot \tau)$

$$y(k) = 2(-1)^{k-1} u(k-1) - 3(-2)^{k-1} u(k-1)$$

$\square 11$

Ex: $F(z) = \frac{z(z+1)}{(z+2)(z+4)}$ Find $z^{-1} \cdot T$

$$F(z) = z \left[\frac{(z+1)}{(z+2)(z+4)} \right] = z \left[\frac{A_1}{z+2} + \frac{A_2}{z+4} \right]$$

~~Find~~ $A_1 = \frac{-1}{2}$ $A_2 = \frac{-3}{-2} = \frac{3}{2}$

$$F(z) = z \left[-0.5 \frac{z}{z+2} + 1.5 \frac{z}{z+4} \right]$$

$$F(k) = -0.5 (-2)^k + 1.5 (-4)^k$$

Report solve this problem for $T = 0.5$ sec
 ← الفرض سيكون في آخر خطوة فقط .